Recursion

How TO teach
and NOT TO teach
Recursion
(opinions from the experts?)

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Where do we find materials and examples to present in our lectures?

- Find some really smart people and ask them for help!

Where do we find those really smart people?

Motivated by AP EDG comments:

- …And we teach about real world examples like bank-accounts and employees and Towers-Of-Hanoi…..
- ….. Just say
  - NO to bank accounts ….
  - NO to employees ...
  - NO to Towers-Of-Hanoi?????

Bank Accounts?

- NO to bank accounts?
- What about using bank accounts to talk about recursion? After all…
  - a bank account (the prototypical object) has overdraft protection provided by another account (which may itself have overdraft protection, etc.).

NO to Towers-Of-Hanoi?

- about **162,000** hits for **Towers of Hanoi** on Google!

NO to Towers-Of-Hanoi?

- "…And, yes, please, no bank accounts, no employees, no barking dogs. (But let's keep Towers of Hanoi -- for their sentimental value.)"

- "…I will also echo the pitch to keep Towers of Hanoi, but not for sentimentality. I say keep it because it is a concrete, physical model for recursion that makes sense to kids. They can see and feel the patterns as they move disks, and this makes it pedagogically appropriate."

No to Towers-Of-Hanoi?

- Let me cast a vote for letting Towers of Hanoi finally die. I have never seen this as a helpful example when trying to explain recursion.

Easy to understand?

```
static void towers(char fromPeg, char toPeg, char tempPeg, int
    numDisks)
{
   if (numDisks == 1)
   {
      System.out.println(numDisks + "\t" + fromPeg + "\t"
            + toPeg);
   }
   else
   {
      towers(fromPeg, tempPeg, toPeg, numDisks-1);
      towers(fromPeg, toPeg, tempPeg, 1);
      towers(tempPeg, toPeg, fromPeg, numDisks-1);
   }
```

Towers of Hanoi

- History:
  - http://www.lawrencehallofscience.org/Java/Tower/towerhistory.html
  - http://www.dcs.napier.ac.uk/a.cumming/hanoi/rechelp.html
  - http://www.lhs.berkeley.edu/Java/Tower/index.html
- Explanation of Algorithm
  - http://www.dcs.warwick.ac.uk/~sgm/open/hanoi.html

Towers of Hanoi

- Solution:
  - http://www.cise.ufl.edu/~sahni/dsaaj/JavaVersions/applications/TowersOfHanoi/TowersOfHanoi.htm
  - http://www.ifors.ms.unimelb.edu.au/tutorial/hanoi/
- Problem and Solution
  - http://www.mazeworks.com/hanoi/
  - http://javascript.internet.com/games/hanoi.html
  - http://www.lhs.berkeley.edu/Java/Tower/Tower.html

NO to Towers-Of-Hanoi?

- Personally, I HATE Towers-Of-Hanoi!

  but….

From a workshop participant!

- "At any rate, I'm at a loss--I completely forget the shortcut method for solving the Towers of Hanoi in the least number of moves possible.

- I know it was "_____" then "make a legal move", over and over. I CAN'T REMEMBER THE FIRST PART!!!"

The three rules of Recursion:

Approach every recursive problem as if it were a journey; if you follow these rules, you will complete the journey successfully.

RULE 1: Find out how to take just one step.

RULE 2: Break each journey down into one step plus a smaller journey.

RULE 3: Know when to stop.

After all…Recursive Solutions

are easier to understand than iterative solutions.

Recursive solutions are easier to understand than Iterative solutions.

- Iterative

```
for(i=0; i<=10; i++)
   System.out.println(i);
```

Recursive solutions are easier to understand than Iterative solutions..

- Recursive

```
public static void printNumber(int n)
{
    if (n>=0)
    {
    printNumber(n-1);
    System.out.println(n);
    }
}
```

Recursive solutions are easier to understand than Iterative solutions.

- Iterative

```
public static boolean palindrome(String s)
{
   String reverse = "";
   for(int i = s.length()-1; i>=0; i--)
   {
      reverse  += s.charAt(i);
   }
   return(reverse.equals(s));
}
```

Recursive solutions are easier to understand than Iterative solutions.

- Recursive

```
public static boolean palindrome(String s)
{
   if (s.length() <= 1)
      return true;
   if (s.charAt(0) == s.charAt(s.length()-1))
      return palindrome(s.substring(1, s.length()-1));
   else return false;
}
```

Non-Motivating Examples…

- Fibonacci numbers
- Factorial
- Power
- Combinations C(n,k)

but…

Combinations

```
public static int combo(int amount,int value)
{
    if ((amount < 0) || (value == 0))
        return(0);

    else if (amount == 0)
        return( 1);

    else
     return( combo(amount,value - 1) + combo(amount - value, value));
}
```

Evaluate combo(4,3)

(4, 3)
need help…

Evaluate: `acker(2,3)`

$$\texttt{acker(x, y)} = \begin{cases} \texttt{y + 1} & \texttt{when x = 0} \\ \texttt{acker(x - 1, 1)} & \texttt{when x != 0, y = 0} \\ \texttt{acker(x - 1, acker(x, y - 1))} & \texttt{when x != 0, y != 0} \end{cases}$$

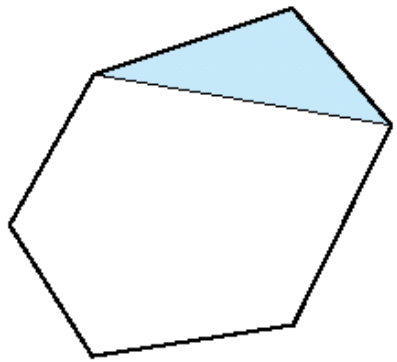
Perhaps more useful but maybe not motivating…

- Print the binary for of a decimal number.
- Return the Greatest Common Factor of two numbers (Euclid's Algorithm).
  - gcd(28, 8) 28 = 3 * 8 + 4
  - gcd(28, 8) = gcd(8, 4)
  - gcd(8, 4) 8 = 2 * 4 + 0
  - gcd(8, 4) = gcd(4, 0)
  - gcd(4, 0) = 4

Area of a Polygon

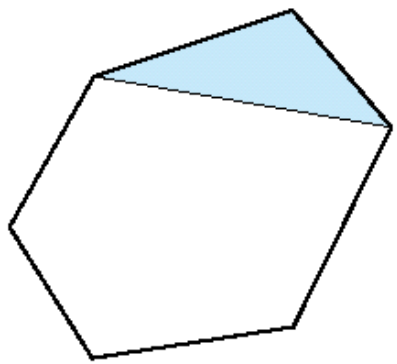
To compute the area of a polygon with more than three corner points:

- chop off a triangle and determine the area of the triangle (take one step).
- compute the area of the remaining polygon (a smaller journey)



Maybe a bit more motivating…

$$A = \frac{\left| x1 * y2 + x2 * y3 + x3 * y1 - y1 * x2 - y2 * x3 - y3 * x1 \right|}{2}$$



Tracing Recursive Problems

- Affectionately known as "mystery methods" in sample AP problems.

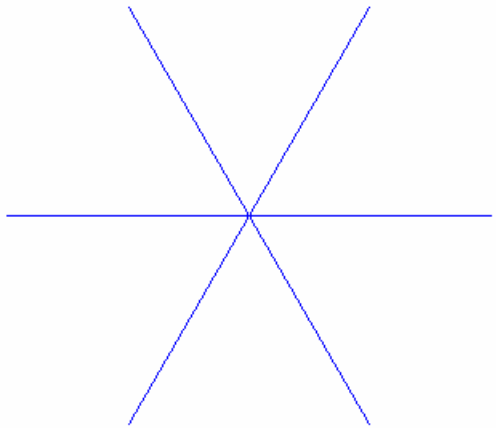
What makes problems motivating?

- Graphics

Motivating example?

- What about a star (really 6 lines)?

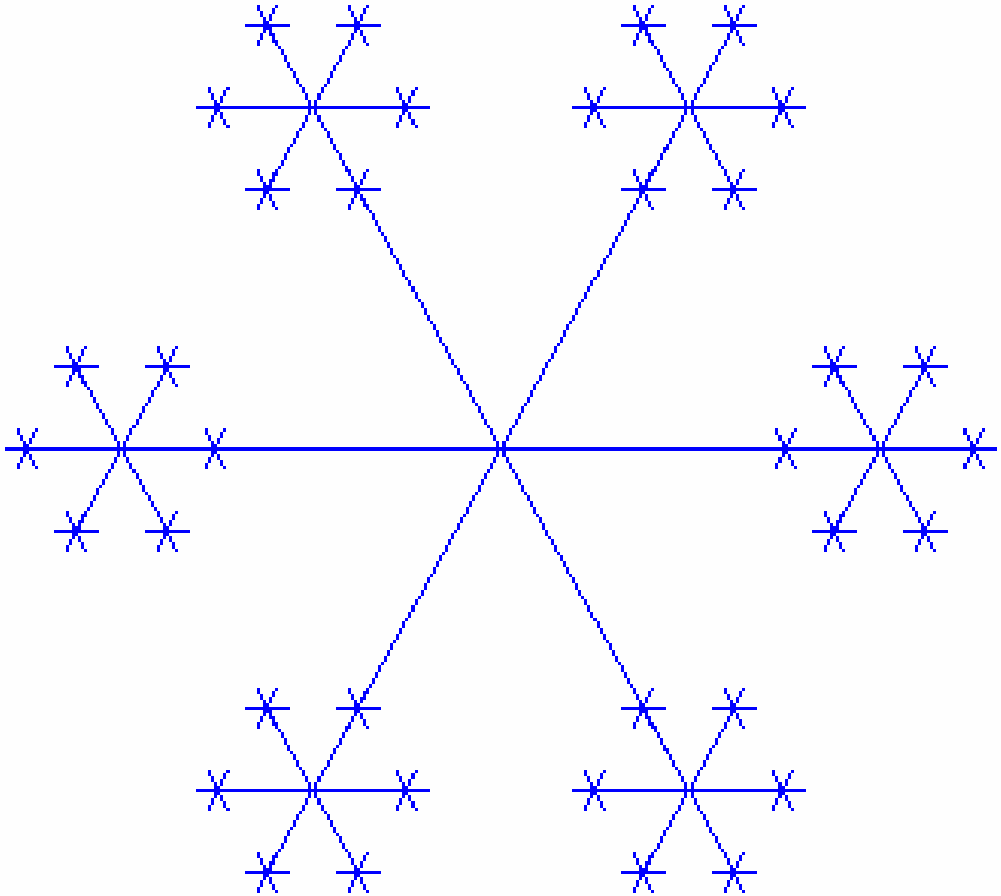
  drawLine(cx, cy, endX, endY)



- Perhaps not so much…

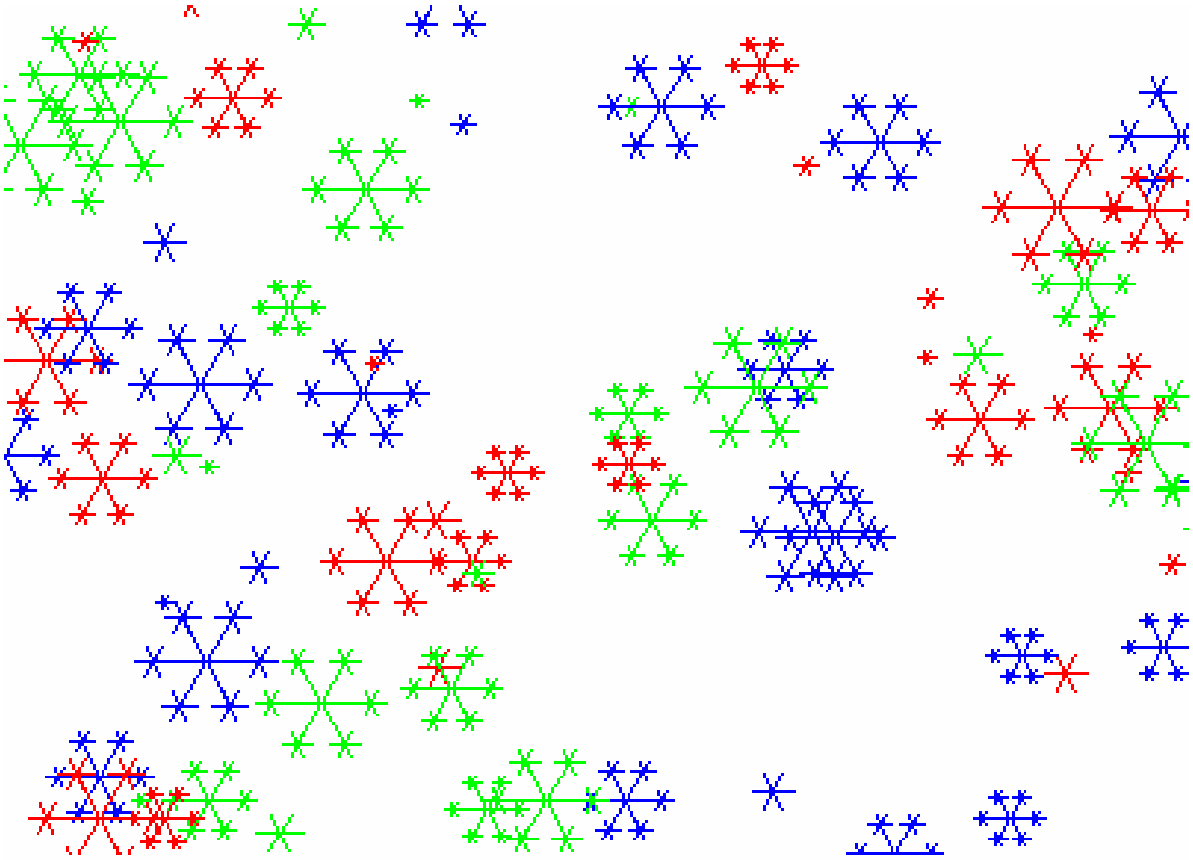
http://chortle.ccsu.ctstateu.edu/CS151/Notes/chap74/ch74_7.html

What about a snowflake?

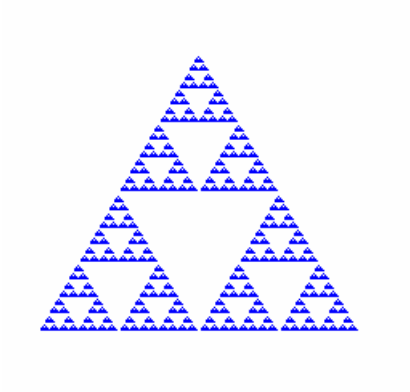


http://chortle.ccsu.ctstateu.edu/CS151/Notes/chap74/ch74_9.html

Or many snowflakes?



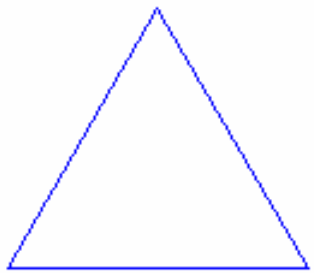
Sierpinski Triangle



- http://www.arcytech.org/java/fractals/sierpinski.shtml
- http://math.bu.edu/DYSYS/chaos-game/node2.html
- http://math.rice.edu/~lanius/fractals/sierjava.html

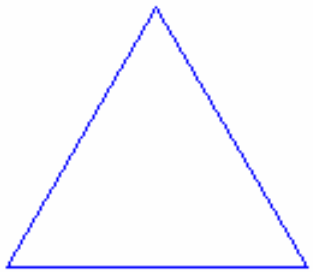
Sierpinski Triangle

- Draw a triangle.
  - How do we do this?



Sierpinski Triangle

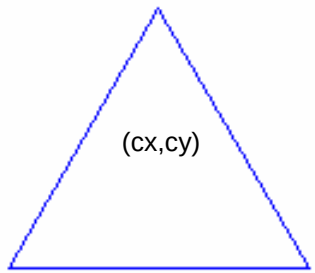
- Now what?



http://www.arcytech.org/java/fractals/sierpinski.shtml

Sierpinski Triangle

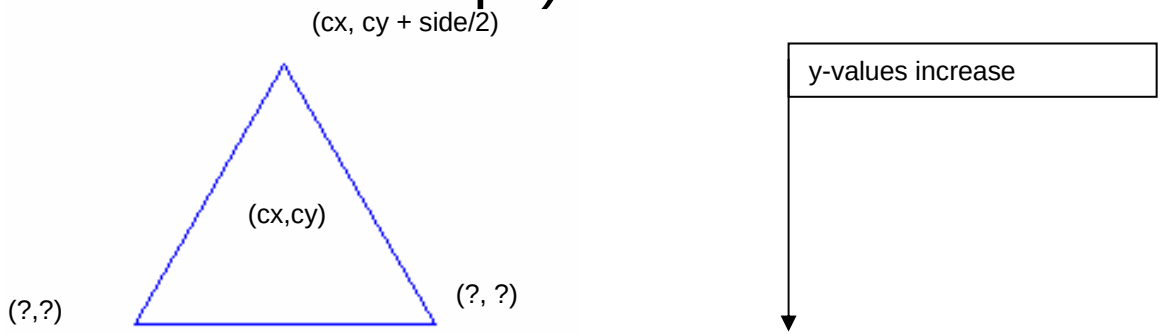
- If the center of this triangle is at (cx,cy), and the length of a side is *side*, find the coordinates of the 3 vertices.



http://www.arcytech.org/java/fractals/sierpinski.shtml

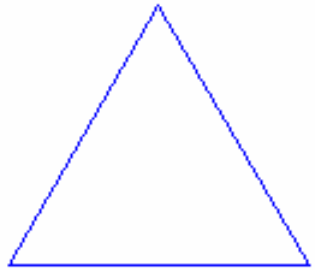
Sierpinski Triangle

- If the center of this triangle is at (cx,cy), and the length of a side is *side*, find the coordinates of the 3 vertices.
  - (Use 30-60-90 relationships)

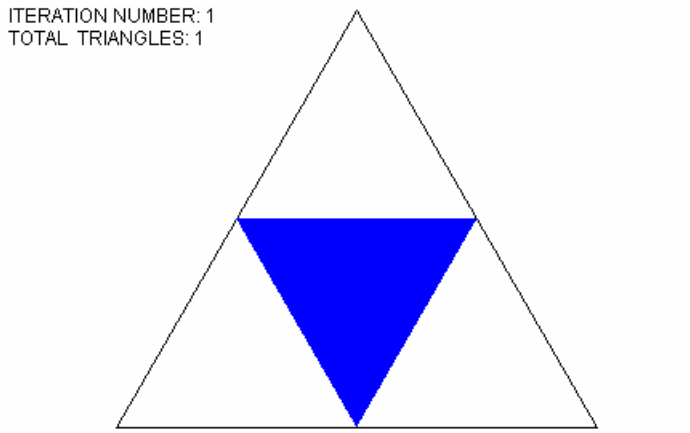


http://www.arcytech.org/java/fractals/sierpinski.shtml

Sierpinski Triangle



- Now what?
  - If the side is small, draw one triangle.
  - If the size is large (not small), draw three triangles that are half the size.

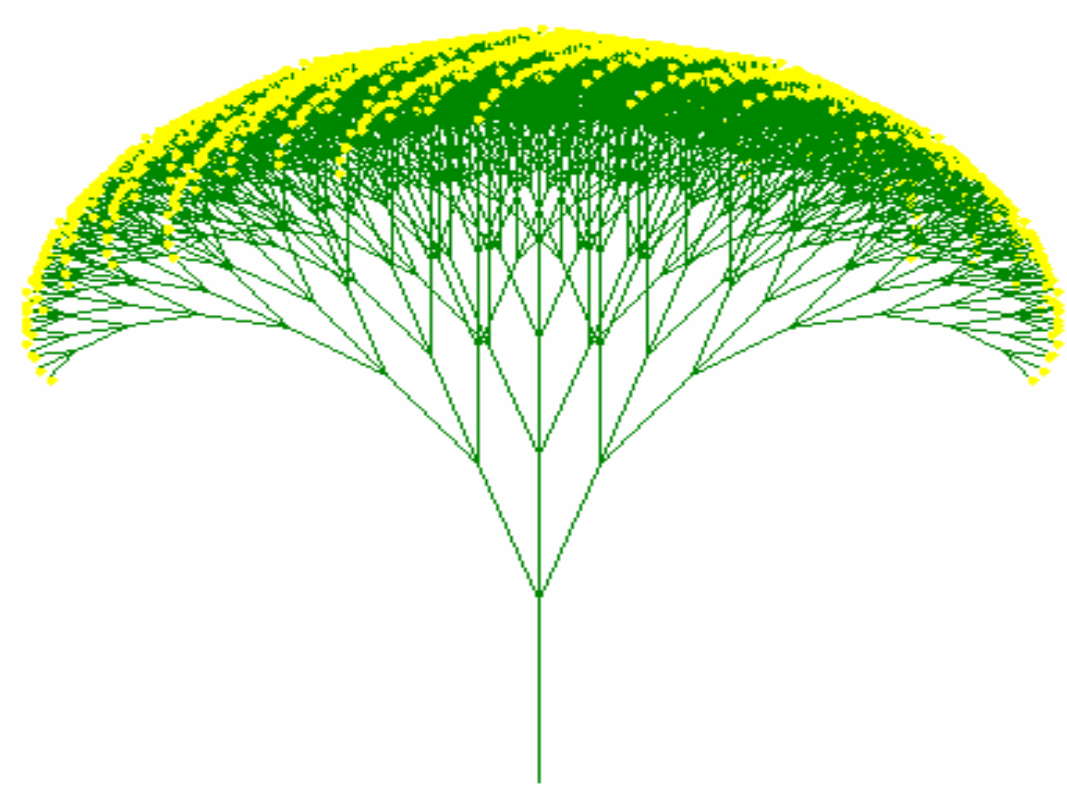


Need more triangle?

- Koch's Snowflake

http://www-cs-faculty.stanford.edu/~eroberts//jtf/demos/KochSnowflakeDemo.html

And of course there's broccoli.



Introducing Recursion

- The Cat in the Hat Comes Back
  - Dr. Seuss

- Martin and the Dragon
  - Touretzky, David S. Touretzky. *Common Lisp: A Gentle Introduction to Symbolic Computation.* (Chapter 8)

Dealing with Infinite Recursion

- Martin and the Dragon
  - Touretzky, David S. Touretzky. *Common Lisp: A Gentle Introduction to Symbolic Computation.* (Chapter 8)

Dealing with Infinite Recursion

- There's a hole in the bucket….
  - http://www.enchbyench.com/angie/dear_liza.htm
  - http://www.songsforteaching.com/folk/theresaholeinthebucket.htm

The ideal educational situation:

- Teach what you need when you need it.

A Story….

- Late one night, a man approaches a street corner and sees another man crawling around on his hands and knees under a lamppost.

  First Man: "What are you doing?"
  Second Man: "Looking for my lost watch. Can you help?"

  So the two of them crawl around and look for awhile, finding nothing.

  First Man: "It doesn't seem to be here. Are you sure this is where you lost it?"

  Second Man: "No. I lost it in that alley over there."

  First Man: "Then why are you looking for it under the lamppost?"

  Second Man: "The light is better here."

  - Posted by Jim Huggins, Wednesday, November 24, 2004 8:47 AM

Thanks to my friends…

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- Owen Astrachan
- Gary Litvin
- Maria Litvin
- Roselyn Teukolsky

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- Carl G. Alphonce
- Greg King

References

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- Roberts, Eric. *Thinking Recursively.*
- http://www.acm.org/education/jtf/
- Bruce, Kim, Andrea Danyluk, and Thomas Murtagh. *Why Structural Recursion should be taught before Arrays in CS1.*
- *http://www.iol.ie/~jmchugh/csc302/*
- http://www.cs.bham.ac.uk/resources/courses/java/msc/lectures/lecture1-2.pdf
- Touretzky, David S. Touretzky. *Common Lisp: A Gentle Introduction to Symbolic Computation.* (Chapter 8) http://www-2.cs.cmu.edu/~dst/LispBook/
- Trees, Frances P. and Cay Horstmann. Computing Concepts with Java Essentials AP CS Student Guide.

Finally….

- In order to understand recursion you must understand recursion.

Dan R. Ghica, *Java Workshop: Recursion*

http://www.cs.bham.ac.uk/resources/courses/java/msc/lectures/lecture1-2.pdf