Counting in Different Number Systems

Base 10 (Decimal) is important **because** that is the base that we first learn in our culture.

Base 2 (Binary) is important **because** that is the base used for computer codes and instructions. To "understand how a computer thinks", you must "think in binary".

Bases 8 (Octal) and 16 (Hexadecimal) are important **because** they are easier to convert to binary than is decimal (because they are powers of 2) and because they suffer fewer typo-type errors than binary (because the same value requires fewer digits to copy and manipulate than in binary).

Today computer programmers "talk decimal", "work hex" and "think binary".

Talk Decimal	Think Binary	Work Octal	Work Hexadecimal
10 Distinct Digits	2 Distinct Digits	8 Distinct Digits	16 Distinct Digits
Calc Groups of 10	Calc Groups of 2	Calc Groups of 8	Calc Groups of 16
0	0000 0000	0	0
1	0000 0001	1	1
2	0000 0010	2	2
3	0000 0011	3	3
4	0000 0100	4	4
5	0000 0101	5	5
6	0000 0110	6	6
7	0000 0111	7	7
1X = 1 Group of 10	Groups of Power of 2	1X = 1 Group of 8	1X = 1 Group of 16
8	0000 1000	10	8
9	0001 0001	11	9
10	0001 0010	12	A
11	0001 0011	13	B
12	0001 0100	14	C
13	0001 0101	15	D
14	0001 0110	16	E
15	0001 0111	17	F

The position of a digit determines its magnitude == multiply by some power of the base.

Talk Decimal	Think Binary	Work Octal	Work Hexadecimal
10 Distinct Digits	2 Distinct Digits	8 Distinct Digits	16 Distinct Digits
Calc Groups of 10	Calc Groups of 2	Calc Groups of 8	Calc Groups of 16
0 x 10^0	0 0 0 0 0 0 0 0 128 64 32 16 8 4 2 1	0 x 8^0	0 x 16^0
1 x 10^0	0 0 0 0 0 0 0 1 128 64 32 16 8 4 2 1	1 x 8^0	1 x 16^0
2 x 10^0	0 0 0 0 0 0 1 0 128 64 32 16 8 4 2 1	2 x 8^0	2 x 16^0
3 x 10^0	0 0 0 0 0 0 1 1 128 64 32 16 8 4 2 1	3 x 8^0	3 x 16^0
4 x 10^0	0 0 0 0 0 1 0 0 128 64 32 16 8 4 2 1	4 x 8^0	4 x 16^0
5 x 10^0	0 0 0 0 0 1 0 1 128 64 32 16 8 4 2 1	5 x 8^0	5 x 16^0
6 x 10^0	0 0 0 0 0 1 1 0 128 64 32 16 8 4 2 1	6 x 8^0	6 x 16^0
7 x 10^0	0 0 0 0 0 1 1 1 128 64 32 16 8 4 2 1	7 x 8^0	7 x 16^0
1X = 1 Group of 10	Groups of Power of 2	1X = 1 Group of 8	1X = 1 Group of 16
8 (0 x 10^1 + 8 x 10^0)	0 0 0 0 1 0 0 0 128 64 32 16 8 4 2 1	10 = 8 + 0 (1 x 8^1 + 0 x 8^0)	8 (0 x 16^1 + 8 x 16^0)
9 (0 x 10^1 + 9 x 10^0)	0 0 0 0 1 0 0 1 128 64 32 16 8 4 2 1	11 = 8 + 1 (1 x 8^1 + 1 x 8^0)	9 (0 x 16^1 + 8 x 16^0)
10 (1 x 10^1 + 0 x 10^0)	0 0 0 0 1 0 1 0 128 64 32 16 8 4 2 1	12 = 8 + 2 (1 x 8^1 + 2 x 8^0)	A $= 10_{10}$ (0 x 16^1 + 10 x 16^0)
11 (1 x 10^1 + 1 x 10^0)	0 0 0 0 1 0 1 1 128 64 32 16 8 4 2 1	13 = 8 + 3 (1 x 8^1 + 3 x 8^0)	B $= 11_{10}$ (0 x 16^1 + 11 x 16^0)
12 (1 x 10^1 + 2 x 10^0)	0 0 0 0 1 1 0 0 128 64 32 16 8 4 2 1	14 = 8 + 4 (1 x 8^1 + 4 x 8^0)	C $=12_{10}$ (0 x 16^1 + 12 x 16^0)
13 (1 x 10^1 + 3 x 10^0)	0 0 0 0 1 1 0 1 128 64 32 16 8 4 2 1	15 = 8 + 5 (1 x 8^1 + 5 x 8^0)	D $= 13_{10}$ (0 x 16^1 + 13 x 16^0)
14 (1 x 10^1 + 4 x 10^0)	0 0 0 0 1 1 1 0 128 64 32 16 8 4 2 1	16 = 8 + 6 (1 x 8^1 + 6 x 8^0)	E $= 14_{10}$ (0 x 16^1 + 14 x 16^0)
15 (1 x 10^1 + 5 x 10^0)	0 0 0 0 1 1 1 1 128 64 32 16 8 4 2 1	17 = 8 + 7 (1 x 8^1 + 7 x 8^0)	F $= 15_{10}$ (0 x 16^1 + 15 x 16^0)

Converting Other Bases To Decimal

The position of a digit determines its magnitude == multiply by some power of the base.

[Remember: Any value to the power of zero is equal to the value of one. $X^0 = 1$]

$2735_{10} = 2 \times 10^3 + 7 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$
$= 2 \times 1000 + 7 \times 100 + 3 \times 10 + 5 \times 1$
$= 2000 + 700 + 30 + 5$
$= 2{,}735_{10}$

$2735_8 = 2 \times 8^3 + 7 \times 8^2 + 3 \times 8^1 + 5 \times 8^0$
$= 2 \times 512 + 7 \times 64 + 3 \times 8 + 5 \times 1$
$= 1024 + 448 + 24 + 5$
$= 1{,}501_{10}$

$2735_{16} = 2 \times 16^3 + 7 \times 16^2 + 3 \times 16^1 + 5 \times 16^0$
$= 2 \times 65{,}536 + 7 \times 256 + 3 \times 16 + 5 \times 1$
$= 131{,}072 + 1{,}792 + 48 + 5$
$= 132{,}917_{10}$

$2735_2 =$ **ILLEGAL because base 2 only has the digits zero and one [0 and 1] ≠ 2 or 7 or 3 or 5 !!**

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$1011_{10} = 1 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 1 \times 10^0$
$= 1 \times 1000 + 0 \times 100 + 1 \times 10 + 1 \times 1$
$= 1000 + 0 + 10 + 1$
$= 1{,}011_{10}$

$1011_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$
$= 8 + 0 + 2 + 1$
$= 11_{10}$

$1011_8 = 1 \times 8^3 + 0 \times 8^2 + 1 \times 8^1 + 1 \times 8^0$
$= 1 \times 512 + 0 \times 64 + 1 \times 8 + 1 \times 1$
$= 512 + 0 + 8 + 1$
$= 521_{10}$

$1011_{16} = 1 \times 16^3 + 0 \times 16^2 + 1 \times 16^1 + 1 \times 16^0$
$= 1 \times 65{,}536 + 0 \times 256 + 1 \times 16 + 1 \times 1$
$= 65{,}536 + 256 + 16 + 1$
$= 65{,}809_{10}$

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$5B7_{16} = 5 \times 16^2 + 11 \times 16^1 + 7 \times 16^0$
$= 5 \times 256 + 11 \times 16 + 7 \times 1$
$= 1{,}280 + 176 + 7$
$= 1{,}463_{10}$

Every octal digit corresponds precisely to three binary digits. 472_8

4_8	7_8	2_8
100_2	111_2	010_2

Every hex digit corresponds precisely to four binary digits. $F5A_{16}$

F_{16}	5_{16}	A_{16}
1111_2	0101_2	1010_2

Conversion	Strategy
Octal to Binary	From right to left, convert each octal digit to three binary digits.
Hexadecimal to Binary	From right to left, convert each octal digit to four binary digits.
Binary to Octal	From right to left, convert each group of three bits to one octal digit.
Binary to Hexadecimal	From right to left, convert each group of four bits to one hex digit.
Octal to Hexadecimal	Convert octal to binary, then binary to hexadecimal.
Hexadecimal to Octal	Convert hexadecimal to binary, then binary to octal.
Binary to Decimal	http://www.wikihow.com/Convert-from-Binary-to-Decimal http://www.newton.dep.anl.gov/newton/askasci/1995/math/MATH065.HTM
Decimal to Binary	http://www.wikihow.com/Convert-from-Decimal-to-Binary
Octal to Decimal	See above: Converting other bases to Decimal.
Decimal to Octal	
Hex to Decimal	See above: Converting other bases to Decimal.
Decimal to Hex	

Subtraction In The Base 16: **Example:** X_{16} = FEED$_{16}$ - 6ACE$_{16}$

First jot this down: **A=10** **B=11** **C=12** **D=13** **E=14** **F**=15

Keep the digits of the same magnitude in the same columns.

F_{16}	E_{16}	E_{16}	D_{16}
6_{16}	A_{16}	C_{16}	E_{16}

Convert base 16 digits to decimal values because that is the base that you think with.

15	14	14	13
6	10	12	14

Write result by column following same rules as decimal subtraction, but adjust for hexadecimal.
When borrowing 1 group from the next column, borrow a group of 16, not 10 (base is 16).

15	14	**Borrow 1 group of 16: ~~14~~ 13**	**13 + 16 = 29**
6	10	12	14

Subtract, keeping results in the correct column.

15	14	13	29
6	10	12	14
9_{10}	**4_{10}**	**1_{10}**	**15_{10}**

Convert results column by column back to hexadecimal digits.

9_{16}	4_{16}	1_{16}	F_{16}

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Solve for Unknown Value X in Different Bases: **Example:** X_{16} = 3676$_8$

The word "bit" is derived from the phrase "binary digit". A "bit" is the digit one (1) or zero (0).

Start With Base 16 Value	"="	Start With Base 8 Value
X_{16}	"="	3 6 7 6
	"="	011 110 111 110 Group Octal as 3 bits per octal digit.
	"="	0111 1011 1110 Group Hex as 4 bits per hex digit.
	"="	7_{10} 11_{10} 14_{10} Convert Bits to Decimal digits.
	"="	**Recall Base 16: A=10 B=11 C=12 D=13 E=14 F=15**
	"="	7_{16} B_{16} E_{16} Result is now in Base 16

Solve for Unknown Value X in Different Bases: Example: $X37_8 = 1XF_{16}$

First jot this down: **A=10** **B=11** **C=12** **D=13** **E=14** **F=15**

Start With Base 8 Value	"="	Start With Base 16 Value
$X\,3\,7_8$	"="	$1\,X\,F_{16}$
$X \times 8^2 + 3 \times 8^1 + 7 \times 8^0$	"="	$1 \times 16^2 + X \times 16^1 + F \times 16^0$
X x 64 + 3 x 8 + 7 x 1	"="	1 x 256 + X x 16 + 15 x 1
64X + 24 + 7	"="	256 + 16X + 15
64X - 16X	"="	256 + 15 - 24 - 7
48X	"="	240
X	"="	240 / 48
X	"="	5
$\mathbf{5}\,3\,7_8$	"="	$1\,\mathbf{5}\,F_{16}$

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Solve for Unknown Value X in Different Bases: Example: $47X_8 = 1XB_{16}$

First jot this down: **A=10** **B=11** **C=12** **D=13** **E=14** **F=15**

Start With Base 8 Value	"="	Start With Base 16 Value
$4\,7\,X_8$	"="	$1\,X\,B_{16}$
$4 \times 8^2 + 7 \times 8^1 + X \times 8^0$	"="	$1 \times 16^2 + X \times 16^1 + 11 \times 16^0$
4 x 64 + 7 x 8 + X x 1	"="	1 x 256 + X x 16 + 11 x 1
256 + 56 + X	"="	256 + 16X + 11
X - 16X	"="	256 + 11 - 256 - 56
- 15X	"="	- 45
X	"="	- 45 / - 15
X	"="	3
$4\,7\,\mathbf{3}_8$	"="	$1\,\mathbf{3}\,B_{16}$

This document was developed to help students prepare for the ACSL Contest #1 of 2007/08.
See: http://www.comscigate.com/HW/cs302/acsl/acsl.htm

Links to further resources: http://www.comscigate.com/numSystems/numSys.htm